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Anisotropy in the dynamic non-linear viscoelastic properties of bovine compact bone

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A non-linear model is proposed to characterize the mechanical behaviour of compact bone and has been validated by mechanical tests on bovine femoral compact bone. The model consists of a non-linear spring and a non-linear dashpot in parallel. The method of identifying the viscoelastic parameters in the model is discussed. Impact and quasi-static compressive mechanical tests have shown that the model can be applied to bone to represent its mechanical behaviour at strain rates up to $100\ \mathrm{s}^{-1}$. The directional dependence of the viscoelastic parameters for the bone are determined and discussed.

1. Introduction

In order to develop bone-analogue materials which combine suitable mechanical compatibility with a favourable bioactive response, the evaluation of the mechanical properties of bone is necessary [1]. Also from a clinical viewpoint, for the prediction of injury or fracture it is important to establish the mechanical model or constitutive equation applicable to bone over a wide range of loading rates since bone fracture may easily occur in high-speed accidents or even in a slow fall. Furthermore, anisotropy in mechanical properties due to the microstructure has to be considered. Many investigations concerning the mechanical properties of compact bone have been reported in the field of biomechanics [2–4] but work on its dynamic response in various orientations over a wide range of loading rates is limited.

This paper deals with the anisotropy in the dynamic non-linear viscoelastic behaviour of bovine femoral compact bone. The objectives can be explicitly stated as:

1. To establish the mechanical model (constitutive equation) to represent the non-linear stress–strain characteristics of the bone both under impact and quasi-static compressive loading;
2. To develop a method for the identification of determining the unknown parameters in the model; and
3. To determine the orientational dependence of the viscoelastic characteristics such as the stress–strain relationship, stiffness and damping loss.

2. Procedure for the identification of viscoelastic characteristics

The identification of the viscoelastic characteristics is based on the stress–strain relations obtained by impact and quasi-static tests and needs the specific constitutive equation to express the non-linear viscoelastic behaviour. Hence we introduce the following functional formula to represent bone's mechanical behaviour:

$$\sigma = f(\varepsilon, \dot{\varepsilon}, \boldsymbol{P}) \qquad (1)$$

where

σ = longitudinal stress in the specimen
ε = longitudinal strain in the specimen
$\dot{\varepsilon}$ = strain rate
$\boldsymbol{P}$ = viscoelastic parameters.

Once we have obtained the stress–strain relationship under impact loading, the parameter values in Equation 1 can be identified by solving a minimization problem (Equation 2) by the method of least squares. A detailed description of the solution procedure can be found elsewhere [5].

$$\min_{P} F = \min_{P} \left\{ \sum_{k=1}^{N} [\sigma_T(\varepsilon_k, \dot{\varepsilon}_k, \boldsymbol{P}) - \sigma_m(\varepsilon_k, \dot{\varepsilon}_k)]^2 \right\} \qquad (2)$$

where

$F = \sum_{k=1}^{N} [\sigma_T(\varepsilon_k, \dot{\varepsilon}_k, \boldsymbol{P}) - \sigma_m(\varepsilon_k, \dot{\varepsilon}_k)]^2$; the least squares distance function
σ_T = theoretical stress at given strain (ε_k; $k = 1, \ldots, N$) and strain rate ($\dot{\varepsilon}_k$; $k = 1, \ldots, N$)
σ_m = stress measured at given strain (ε_k; $k = 1, \ldots, N$) and strain rate ($\dot{\varepsilon}_k$; $k = 1, \ldots, N$)
N = number of data points.

3. Materials

All specimens were fabricated from batches of fresh frozen bovine femurs of an average of seven or eight

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months. Bone samples were machined from the dense cortical material of the posterior part of the femoral mid-diaphysis into cylinders 5–10 mm in diameter and 5–10 mm long but with a constant length to diameter ratio of 1. Since mechanical properties were determined as a function of direction, a reference system had to be chosen for the orientation of the specimens in the cortical bone. The Cartesian coordinate system shown in Fig. 1 was chosen. The bone axis is parallel to the long axis of the bone, the tangential axis is in the circumferential direction while the radial axis is in the endosteal–periosteal direction. These axes were specified by eye and denoted BA, TA and RA, respectively. Specimens were cut with their long axis in the BA–TA plane, TA–RA plane and RA–BA plane at angles of 0°, 15°, 30°, 45°, 60°, 75° and 90° to the BA, TA and RA axes, respectively. After sectioning, specimens were stored in saline solution at 4 °C for up to 12 h before testing. All specimens showed secondary Haversian systems (Haversian bone). In this study, only one specimen at each orientation was taken from each femur and five specimens were tested for each orientation.

4. Experimental method

Quasi-static compression tests, below the yield stress, were performed at a strain rate of $10^{-3}\,s^{-1}$ using a Shimadzu AG-25TD materials testing machine to obtain the static stress–strain relations. Impact compression tests on the same specimens were then carried out at strain rate of approximately $100\,s^{-1}$ using the split-Hopkinson pressure-bar (SHPB) technique to obtain the dynamic stress–strain relations.

Fig. 2 shows the schematic diagram of the SHPB apparatus used in this study. The apparatus consists of an air gun, a striker bar, two Hopkison pressure bars (one input and one output bar) and recording equipment. The specimen was held in place between the input and output bars. During testing a lubricant was placed on the contact surfaces to reduce the frictional constraint and to ensure contact at the interfaces, permitting stress wave transmission. The incident and transmitted stress waves were detected by foil strain gauges bonded to the input and output bars, respectively. Signals of these waves were stored in a digital storage oscilloscope with a sampling frequency of 4 MHz. Subsequent analysis of the recorded data was performed using a personal computer. Typical oscilloscope records for the SHPB test on the specimen oriented parallel to the BA-direction are shown in Fig. 3. Strain rate ($\dot{\varepsilon}$), strain (ε) and stress (σ) at any time (τ) in the specimen were evaluated by the following equations [6].

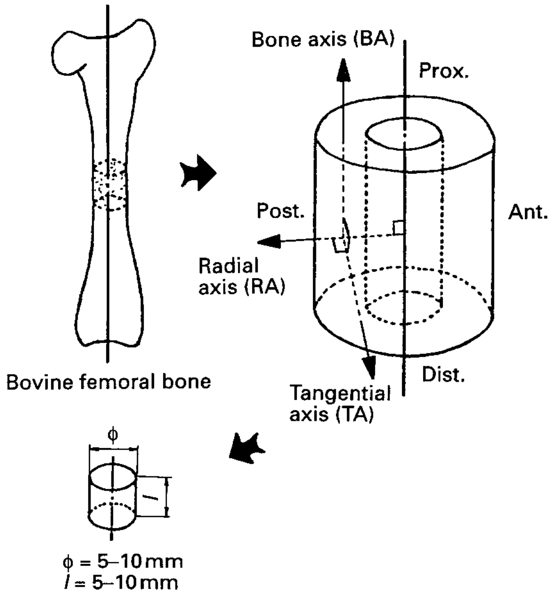

Figure 1 Cartesian coordinate system chosen for compact bone specimens.

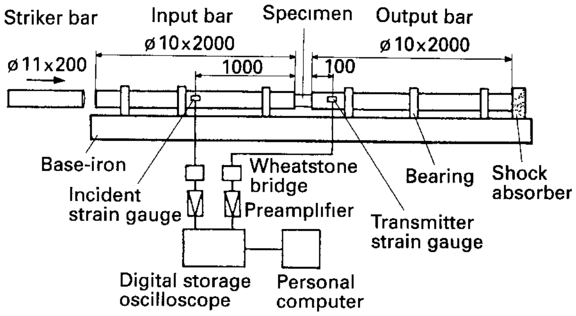

Figure 2 Split-Hopkinson pressure-bar apparatus.

$$\dot{\varepsilon} = \frac{2}{\rho_0 c_0 l}(\sigma_I - \sigma_T) \tag{3}$$

$$\varepsilon = \int_0^\tau \dot{\varepsilon}\,dt = \frac{2}{\rho_0 c_0 l}\int_0^\tau (\sigma_I - \sigma_T)\,dt \tag{4}$$

$$\sigma = \frac{A}{A_0}\sigma_T \tag{5}$$

where subscripts I and T refer to the incident and transmitted stress waves, respectively, and

l = initial length of the specimen
ρ_0 = density of the input and output bars
c_0 = longitudinal elastic wave velocity of the input and output bars
A = cross-sectional area of the specimen and
A_0 = cross-sectional area of the input and output bars.

All quasi-static and impact tests were performed at room temperature (20 °C) and the specimens were kept moist during testing.

5. Results and discussion

5.1. Stress–strain relationship

Fig. 4 shows typical stress–strain curves under quasi-static compression in the BA-, TA- and RA-directions. The stress–strain relationships are approximately linear and the stiffness in the BA-direction is greater than that in the other two directions.

Fig. 5 shows the equivalent stress–strain curves under impact compression. These curves obviously show non-linear characteristics. The stiffness is higher in the RA-direction and highest in the BA-direction. It is obvious from Figs 4 and 5 that the non-linearity in the stress–strain relationship becomes more significant at higher loading rates.

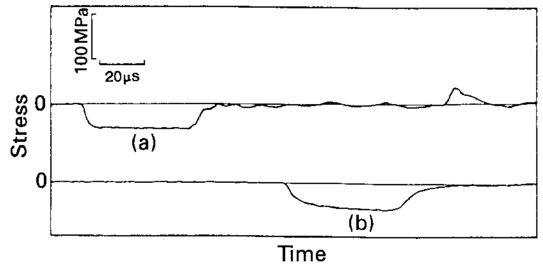

Figure 3 Typical oscilloscope records for SHPB test: (a) incident stress pulse in input bar (downward is compression); (b) transmitted stress pulse in output bar (downward is compression). Specimen axis: BA-direction.

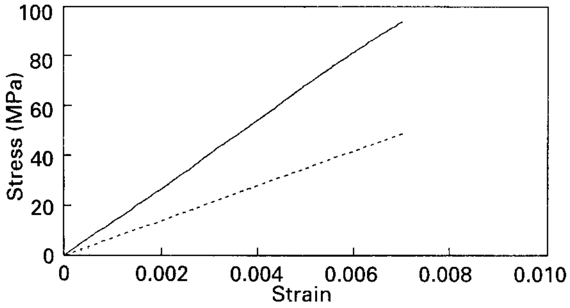

Figure 4 Stress–strain curves at various directions under quasi-static compression (—— BA-direction; – – – RA-direction; . . . . . TA-direction).

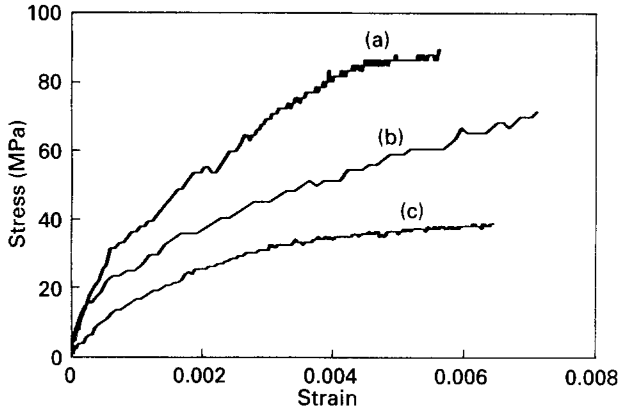

Figure 5 Stress–strain curves at various directions under impact compression: (a) BA-direction; (b) RA-direction; (c) TA-direction.

5.2. Viscoelastic properties

Figs 4 and 5 emphasize that the non-linear viscoelastic model is necessary to represent the mechanical behaviour of bone. Hence the model with a non-linear spring and non-linear dashpot in parallel, as shown in Fig. 6, was adopted in this study. The constitutive equation is given in Equation 6

$$\sigma = E\varepsilon - \alpha\varepsilon^2 + \mu\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)^p \qquad (6)$$

where

E, α, μ, p = viscoelastic constants as shown in Fig. 6

$\dot{\varepsilon}_0 = 1.0\,\mathrm{s}^{-1}$.

The non-linear viscoelastic parameter values, i.e. $\boldsymbol{P} = (E, \alpha, \mu, p)$, were determined based on the stress–strain curves under impact compression using the previously described method. This model was validated by plotting the experimental stress–strain curves and the theoretical curves computed by Equation 6 with the identified parameter values of E, α, μ and p as shown in Fig. 7. The model with the identified parameter values yields fairly good prediction of the stress–strain relations both under impact and quasi-static compression. The directional dependent behaviour of the viscoelastic characteristics was examined based on these parameter values, E, α, μ and p, identified at each direction.

lus E and α (which is related to the order of magnitude in the non-linearity in stress–strain relationship) with respect to orientation of specimen axis, respectively. Quasi-static modulus, E_s, is also plotted against orientation in Fig. 8. Both E and E_s in directions up to 30° from the BA axis in the BA–TA plane are greater than those in other directions. A similar trend can be seen in the variation of α (Fig. 9). These results suggest that the rigidity as well as non-linearity in stress–strain response of the bone, both under impact and quasi-static loading, is greatest up to 30° from the BA axis in the BA–TA plane. This is probably due to the direction of the lamellae of the Haversian systems.

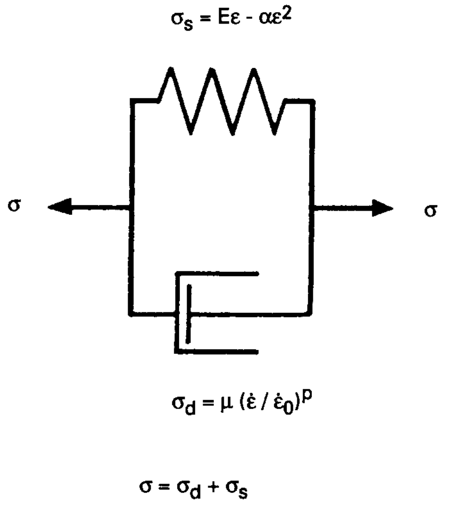

Figure 6 Proposed non-linear viscoelastic model.

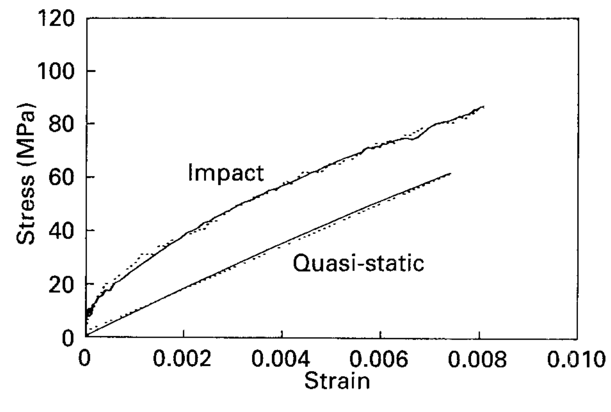

Figure 7 Experimental (. . . .) and theoretical (——) stress–strain curves. The specimen axis was in a direction 15° from the bone axis in the RA–BA plane. Theoretical curves were calculated using Equation 6 with parameter values: E = 9.3 GPa, α = 133.6 GPa, μ = 4.2 MPa and p = 0.30.

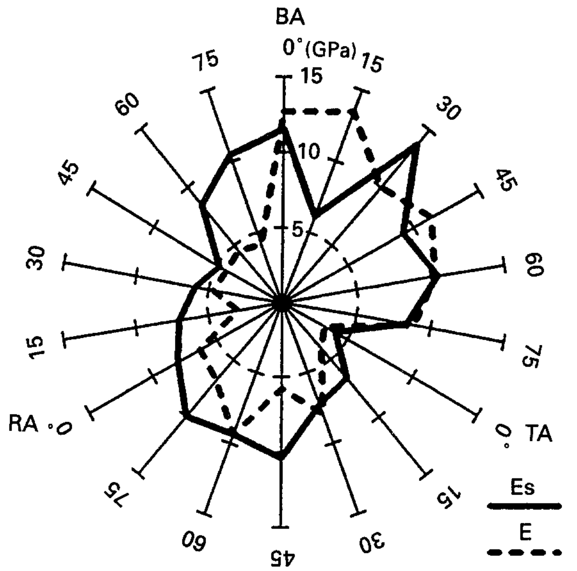

Figure 8 Variation of E and E_s with specimen axis. E_s is Young's modulus under quasi-static compression. Each data point represents the mean of five specimens.

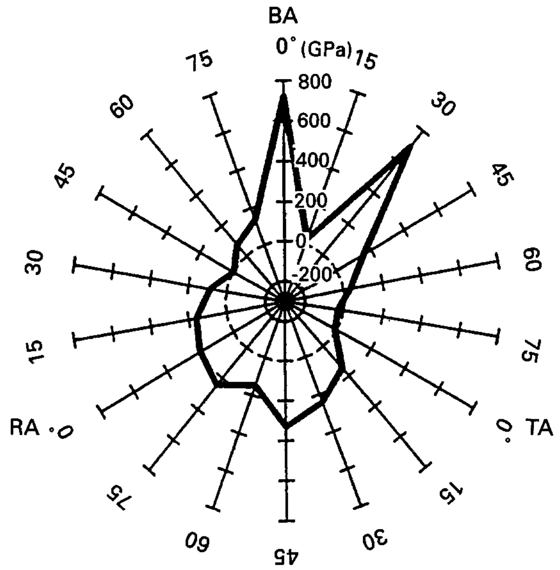

Figure 9 Variation of α with specimen axis. Each data point represents the mean of five specimens.

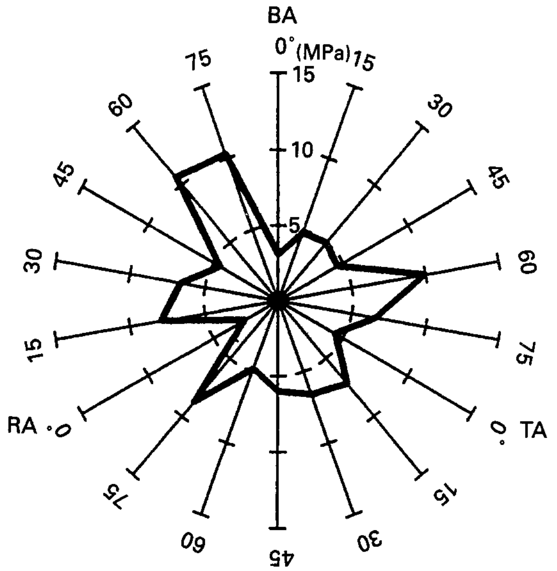

Figure 10 Variation of μ with specimen axis. Each data point represents the mean of five specimens.

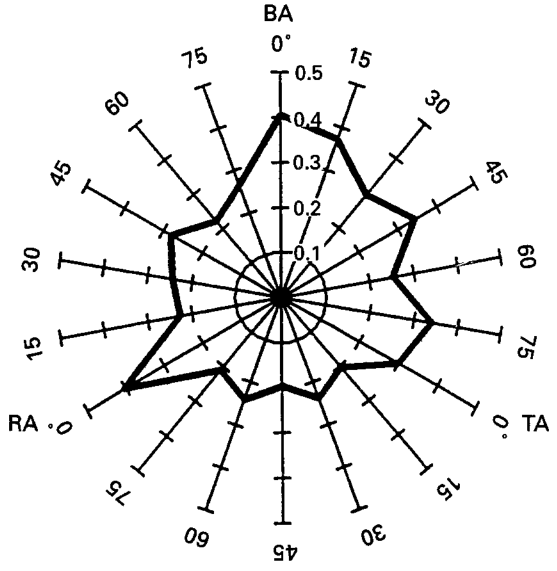

Figure 11 Variation of p with specimen axis. Each data point represents the mean of five specimens.

Figs 10 and 11 show the variation of μ and p with respect to orientation of specimen axis, respectively. Both μ and p are related to the stress due to internal friction, expressed by the third term in Equation 6. μ becomes minimal in the BA-direction (Fig. 10), while p takes a maximum value in the same direction (Fig. 11). Since the stress due to internal friction is more sensitive to the value of p than that of μ (the stress is in proportion to a power of p), it is concluded that friction loss of the bone is highest in the BA-direction.

As a result, the bone is best adapted to absorb impact loads when they are applied along the long axis of the bone.

6. Conclusions

Quasi-static and impact compression tests on bovine femoral compact bone have been performed. A method for the identification of the non-linear viscoelastic properties of the bone has been presented and the effect of orientation on the viscoelastic properties has been studied. The results obtained are summarized as follows:

(1) A non-linear viscoelastic model to represent the mechanical behaviour of bone up to a strain rate of $100\,s^{-1}$ is proposed and its validity is shown through the mechanical tests.

(2) The stiffness, both under impact and quasi-static loading, was greatest at up to 30° from the bone axis in the BA–TA plane.

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